ACTIVITY 13

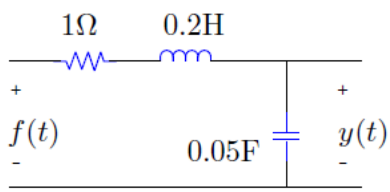
Signals and systems

Dr. Juan Manuel CAMPOS

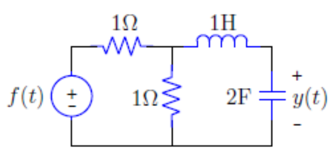
Name____

PART I Frequency response

1. Determine the frequency response $H(\omega) = \frac{Y}{F}$ of the circuit shown and sketch $|H(\omega)|$ versus $\omega \geq 0$. In the diagram, $f(t)$ and $y(t)$ denote the input and output signals of the circuit.



2. In the following circuit, determine the frequency response $H(\omega) = \frac{Y}{F}$ and $H(0)$:



PART II First approach to Fourier Transform

1. a) Given that $f(t) = e^{-a(t-t_o)}u(t-t_o)$, where $a > 0$, determine the Fourier transform $F(\omega)$ of $f(t)$.

   b) Given that

   $$g(t) = \frac{1}{a + jt},$$

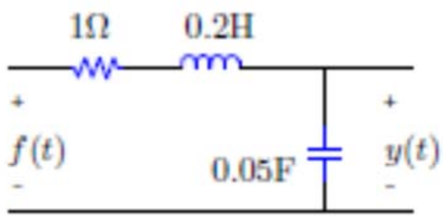
   where $a > 0$, determine the Fourier transform $G(\omega)$ of $g(t)$ by using the symmetry property and the result of part (a).

   c) Confirm the result of part (b) by calculating $g(t)$ from $G(\omega)$ using the inverse Fourier transform integral.

SOLUTION Activity 13

1)

Determine the frequency response $H(\omega) = \frac{Y}{F}$ of the circuit shown and sketch $|H(\omega)|$ versus $\omega \geq 0$. In the diagram, $f(t)$ and $y(t)$ denote the input and output signals of the circuit.



Solution:

Applying voltage division using phasors:

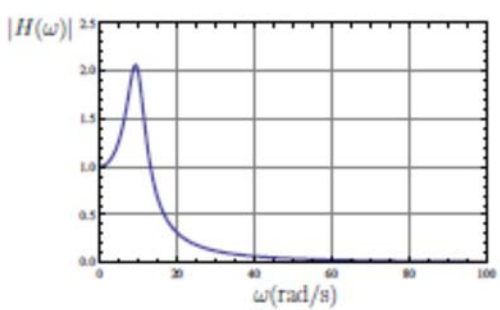
$$\begin{aligned} Y &= F\frac{\frac{1}{j\omega\times 0.05}}{1 + j\omega\times 0.2 + \frac{1}{j\omega\times 0.05}} \\ &= F\frac{1}{1 - 0.01\omega^2 + j\omega 0.05}, \end{aligned}$$

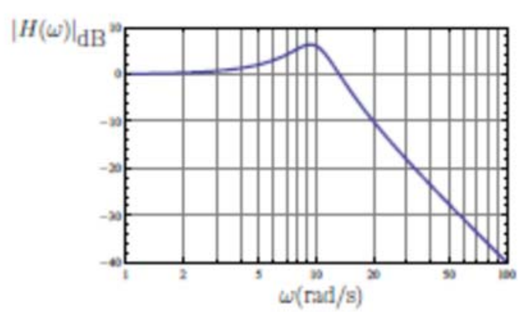
Therefore, the frequency response of the system is

$$H(\omega) = \frac{Y}{F} = \frac{1}{1 - 0.01\omega^2 + j\omega 0.05}.$$

Finally we sketch the amplitude of $H(\omega)$:

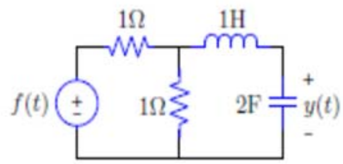
$$|H(\omega)| = \frac{1}{\sqrt{(1 - 0.01\omega^2)^2 + (0.05\omega)^2}}.$$





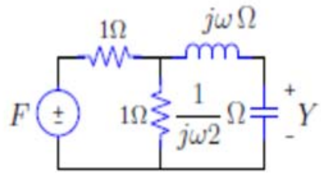
2)

2. In the following circuit, determine the frequency response $H(\omega) = \frac{Y}{F}$ and $H(0)$:

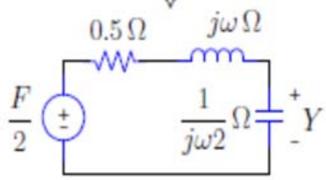


Solution:

The phasor equivalent circuit is as follows:



$\Downarrow$



To get the last simplification we first transformed the F voltage source into a $F/1\Omega$ current source in parallel with a 1Ω resistance, which in parallel with another 1Ωresistance, results in a 0.5Ω resistance. We again perform a source transformation, obtaining a $F \times 0.5$ voltage source in series with a 0.5Ω resistance as depicted in the figure.

Applying voltage division we have

$$Y = \frac{F}{2} \times \frac{\frac{1}{j\omega 2}}{0.5 + j\omega + \frac{1}{j\omega 2}}$$

$$Y = \frac{F}{2 - 4\omega^2 + j\omega 2}.$$

Consequently, the frequency response of the system is

$$H(\omega) = \frac{Y}{F} = \frac{1}{2 - 4\omega^2 + j\omega 2}.$$

Hence,

$$H(0) = \frac{1}{2 - 4(0)^2 + j(0)2} = \frac{1}{2}.$$

3)

a) Given that $f(t) = e^{-a(t-t_o)}u(t-t_o)$, where $a > 0$, determine the Fourier transform $F(\omega)$ of $f(t)$.

b) Given that

$$g(t) = \frac{1}{a+jt},$$

where $a > 0$, determine the Fourier transform $G(\omega)$ of $g(t)$ by using the symmetry property and the result of part (a).

c) Confirm the result of part (b) by calculating $g(t)$ from $G(\omega)$ using the inverse Fourier transform integral.

Solution:

a) Using the Fourier transform formula gives

$$\begin{aligned} F(\omega) &= \int_{t_0}^{\infty} e^{-a(t-t_0)}e^{-j\omega t}dt = \int_{t_0}^{\infty} e^{at_0}e^{-(a+j\omega)t}dt \\ &= \frac{e^{at_0}}{-(a+j\omega)}e^{-(j\omega+a)t}\bigg|_{t_0}^{\infty} = \frac{e^{-j\omega t_0}}{a+j\omega}. \end{aligned}$$

b) From part (a) making $t_0 = 0$, we have

$$f(t) = e^{-at}u(t) \leftrightarrow F(\omega) = \frac{1}{a+j\omega}.$$

Using the symmetry property

$$f(t) \leftrightarrow F(\omega) \Rightarrow F(t) \leftrightarrow 2\pi f(-\omega),$$

we have

$$\frac{1}{a+jt} \leftrightarrow 2\pi e^{a\omega}u(-\omega).$$

Since $g(t) = \frac{1}{a+jt}$, we conclude

$$G(\omega) = 2\pi e^{a\omega}u(-\omega).$$

c) Calculating $g(t)$ from $G(\omega)$,

$$\begin{aligned} g(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)e^{j\omega t}d\omega = \int_{-\infty}^{0} e^{a\omega}e^{j\omega t}d\omega \\ &= \frac{1}{a+jt}\left[e^{(a+jt)\omega}\right]_{-\infty}^{0} = \frac{1}{a+jt},\ a > 0. \end{aligned}$$